

Figure 2. Pressure against molar volume for  $^3\text{He}$ . Curve A, Horner (1970); curve B,  $P_{\text{ISC}}$ , present paper; curve C, Experiment (Dugdale and Franck 1964).

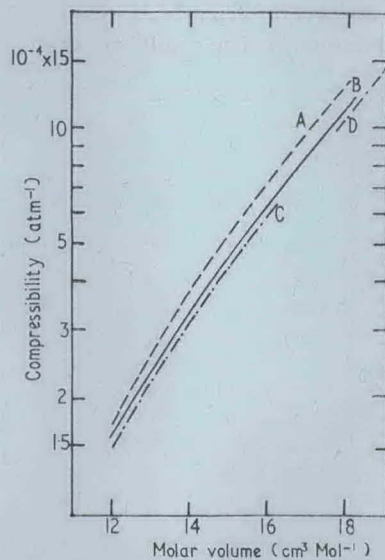


Figure 3. Compressibility against molar volume for  $^3\text{He}$ . Curve A, Horner (1970); curve B,  $\chi_{\text{ISC}}$ , present paper; curve C, Experiment (Dugdale and Franck, 1964); curve D, Experiment (Straty and Adams 1968).

$$F_{\text{sc}} = \frac{1}{2}N \sum_k \langle \phi^k \rangle_{\text{sc}} + \sum_{qs} (f_{qs} - \frac{1}{2}u_{qs}) \quad (1)$$

and

$$\Delta F = - \frac{\hbar^2}{48M^3N} \sum_{1,2,3} \frac{\Delta(q_1 + q_2 + q_3)}{\omega_1\omega_2\omega_3} W_{123} |\Psi(1,2,3)|^2 \quad (2)$$

where

$$W_{123} = (n_1n_2 + n_2n_3 + n_3n_1 + n_1 + n_2 + n_3)(\omega_1 + \omega_2 + \omega_3)^{-1} \\ + 3(n_2n_3 + n_3n_1 - n_1n_2 + n_3)(\omega_1 + \omega_2 - \omega_3)^{-1}$$

and

$$\Psi(1,2,3) = -4i \sum_k \exp \{i(q_1 + q_2 + q_3) \cdot R_k/2\} \sin(\frac{1}{2}q_1 \cdot R_k) \\ \times \sin(\frac{1}{2}q_2 \cdot R_k) \sin(\frac{1}{2}q_3 \cdot R_k) e_{\alpha}(1)e_{\beta}(2)e_{\gamma}(3) \langle \phi_{\alpha\beta\gamma}^k \rangle_{\text{sc}}$$

The ISC free energy,  $F_{\text{ISC}} = F_{\text{sc}} + \Delta F$ .

With these changes the ground state energy, pressure and compressibility of solid  $^3\text{He}$  were calculated in the molar volume range 12 to 18  $\text{cm}^3$ . In the computations it was found necessary to smear up to and including the third neighbours only, the effects of smearing the force constants beyond this point were negligible.  $\Delta F$  was calculated using the frequencies of the all neighbour model but including only nearest neighbours in the lattice sums.

The contribution,  $\Delta P$ , of  $\Delta F$  to the pressure,  $P_{\text{ISC}}$ , and  $\Delta B$ , to the bulk modulus,  $B_{\text{ISC}}$ , and hence the compressibility  $\chi_{\text{ISC}}$ , were determined by numerical differentiation using the expressions

$$P_{\text{ISC}} = - \left( \frac{\partial F_{\text{ISC}}}{\partial V} \right)_T = P_{\text{sc}} + \Delta P$$

and

$$B_{\text{ISC}} = V \left( \frac{\partial^2 F_{\text{ISC}}}{\partial V^2} \right)_T = -V \left( \frac{\partial P_{\text{ISC}}}{\partial V} \right)_T \quad (3) \\ = B_{\text{sc}} + \Delta B = \chi_{\text{ISC}}^{-1}.$$

Subscript ISC refers to quantities calculated using the improved selfconsistent theory.  $P_{\text{sc}}$  was determined from the expression given in equation (2.2) of I, and  $B_{\text{sc}}$  by numerical differentiation of  $P_{\text{sc}}$  using equation (3). The results are shown graphically in figures 1 to 3.

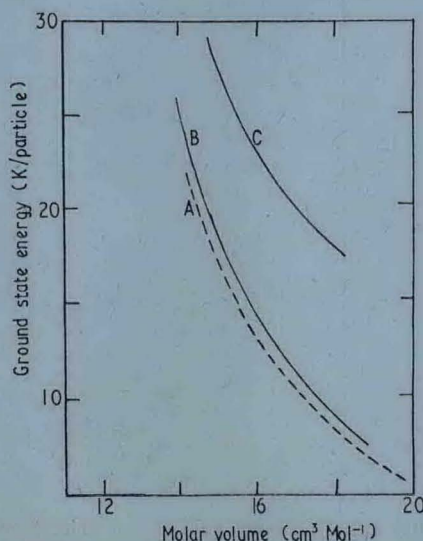


Figure 1. Ground State energy of  $^3\text{He}$  against molar volume. Curve A, Horner, (1970); curve B,  $F_{\text{ISC}} = F_{\text{sc}} + \Delta F$ , present paper; curve C,  $F_{\text{sc}}$ , present paper.